Noise Expressions for Capacitance-Coupled Distributed RC Lines

Hiroshi Kawaguchi and Takayasu Sakurai
Institute of Industrial Science, Univ. of Tokyo,
7-22-1, Roppongi, Minato-ku, Tokyo, 106 Japan
E-mail: kawapy@cc.iis.u-tokyo.ac.jp, tsakurai@iis.u-tokyo.ac.jp

Abstract

Simple yet useful analytical expressions for peak noise amplitude for capacitively coupled two-, three- and infinite interconnections are derived assuming bus lines and other signal lines in VLSI's. The calculated results using the derived formulas are compared with SPICE simulation results to demonstrate the validity of the analytical expressions. Two modes have been studied; the case where adjacent lines are driven from the opposite direction and the case where adjacent lines are driven from the same direction. These cases cover most of the typical situations in VLSI designs and include worst cases in terms of noise amplitude.

In deep submicron VLSI designs where coupling capacitance is comparable to grounding capacitance, the noise induced by the coupling is shown to go up to 40% of the signal swing. This high noise may cause malfunction and timing problems especially in dynamic circuits but even in static circuits, the noise may generate unexpected glitches which may give rise to timing and power problems. The derived expressions are useful in estimating the noise in the early stage of designs and give insight to coupling related issues.

Introduction

In deep submicron designs, interconnection related issues become more and more important in estimating timing behavior of VLSI's [1].

Several attempts have been made to analytically treat the crosstalk in capacitively coupled interconnections. However, the results are limited to two-line systems and the case considered in the previous publications are limited to the case where adjacent lines are driven from the same direction. This paper extends the analysis and covers more general cases. The resultant formulas are more precise than the previously published expressions.*

Notations

Notations used in this paper are as follows.

x : x-coordinate along the line

t : time

r : resistance per unit line length

c : capacitance per unit line length

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* This work is supported by a grant from Toshiba Corporation.

c_c : coupling capacitance per unit line length

l : line length

R : total line resistance (=rl)

C : total line capacitance (=cl)

C_C : total coupling capacitance ($=c_c l$)

R_t : equivalent resistance of driver MOSFET

C_t : equivalent capacitance of receiver MOSFET

R_T : $=R_t/R$

C_T : $=C_t/C$

η : $=C_C/C$

s : Laplace variable

v_i, $v_i(x,t)$: voltage of line i (i=1,2) in t domain

V_i, $V_i(x,s)$: voltage of line i (i=1,2) in s domain

K_1 : residue corresponding to the minimum pole

σ_1 : minimum pole

v_p : peak noise voltage

t_p : time to give the noise peak

p : $=1+(n+1)\eta$

E : step voltage of the driving point

n : number of adjacent lines (n=1 for two-line system, n=2 for three-line system and n=1 for infinite-line systems, see text)

n_1 : $=n+1$

Basic Equations

The basic equations which governs a capacitance-coupled two-line system (Fig. 1) are written as follows.

$$\begin{cases} \dfrac{\partial^2 v_1}{\partial x^2} = r_1(c_1+c_c)\dfrac{\partial v_1}{\partial t} - r_1 c_c \dfrac{\partial v_2}{\partial t} \\ \dfrac{\partial^2 v_2}{\partial x^2} = r_2(c_2+c_c)\dfrac{\partial v_2}{\partial t} - r_2 c_c \dfrac{\partial v_1}{\partial t} \end{cases} \qquad (1)$$

, where r_i and c_i are unit length resistance and capacitance of line i (i=1,2). Since in bus structures and other wiring structures lines have the same unit length resistance and capacitance, we assume $r_1=r_2=r$ and $c_1=c_2=c$.

When three-line system (Fig. 2) is considered, the following equations hold.

$$\begin{cases} \dfrac{\partial^2 v_1}{\partial x^2} = r(c+2c_c)\dfrac{\partial v_1}{\partial t} - 2rc_c \dfrac{\partial v_2}{\partial t} \\ \dfrac{\partial^2 v_2}{\partial x^2} = r(c+c_c)\dfrac{\partial v_2}{\partial t} - rc_c \dfrac{\partial v_1}{\partial t} \end{cases} \qquad (2)$$

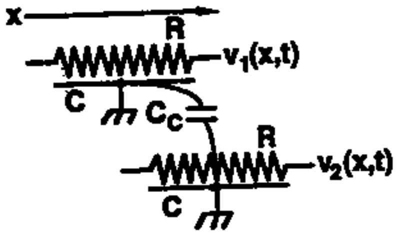

Fig. 1 Capacitively coupled two distributed RC lines.

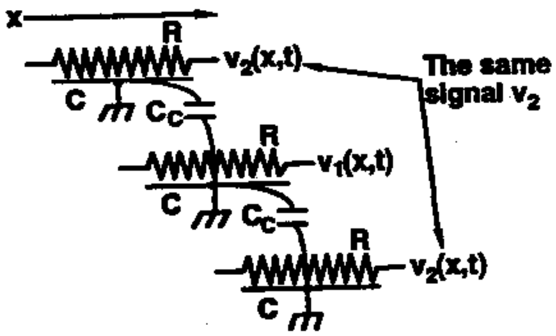

Fig. 2 Capacitively coupled three distributed RC lines.

On the other hand, if infinite number of lines are placed in parallel where the same boundary conditions are applied to every two lines as in Fig. 3, the following equations hold.

$$\begin{cases} \dfrac{\partial^2 v_1}{\partial x^2} = r(c+2c_c)\dfrac{\partial v_1}{\partial t} - 2rc_c\dfrac{\partial v_2}{\partial t} \\ \dfrac{\partial^2 v_2}{\partial x^2} = r(c+2c_c)\dfrac{\partial v_2}{\partial t} - 2rc_c\dfrac{\partial v_1}{\partial t} \end{cases} \quad (3)$$

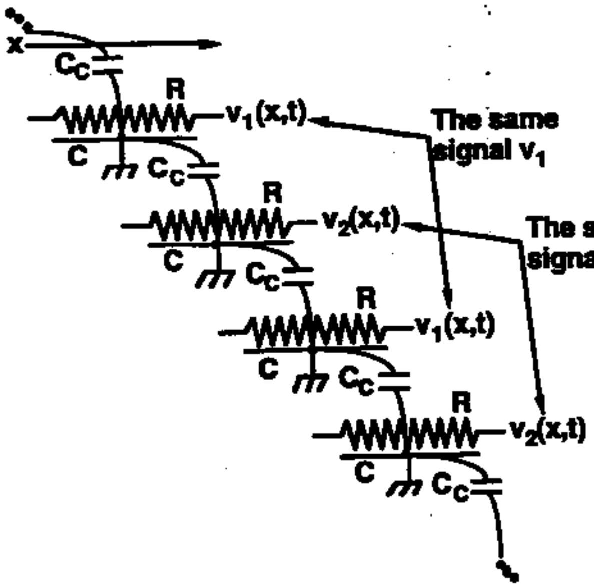

Fig. 3 Capacitively coupled infinite distributed RC lines.

All of the above-mentioned three equation sets can be represented by the following one set of equations.

$$\begin{cases} \dfrac{\partial^2 v_1}{\partial x^2} = r(c+nc_c)\dfrac{\partial v_1}{\partial t} - nrc_c\dfrac{\partial v_2}{\partial t} \\ \dfrac{\partial^2 v_2}{\partial x^2} = r(c+c_c)\dfrac{\partial v_2}{\partial t} - rc_c\dfrac{\partial v_1}{\partial t} \end{cases} \quad (4)$$

In the above Eq. (4), the following values should be set to n and c_c for each case.

Two-line case:	n=1, c_c=c_c
Three-line case:	n=2, c_c=c_c
Infinite number of lines:	n=1, c_c=$2c_c$

Eq. (4) can be simplified, if the following substitutions are made.

$$c_c/c \to \eta,\ \frac{\partial^2 v}{\partial x^2} \to v'',\ \frac{\partial v}{\partial \tau} \to \dot{v}$$

$$\begin{cases} v_1'' = (1+n\eta)\dot{v}_1 - n\eta\dot{v}_2 \\ v_2'' = (1+\eta)\dot{v}_2 - \eta\dot{v}_1 \end{cases}$$

are the resultant equations after the substitution. With a linear transformation, we have

$$\begin{cases} (v_1 - v_2)'' = (1+(n+1)\eta)(\dot{v}_1 - \dot{v}_2) \\ (v_1 + nv_2)'' = \dot{v}_1 + n\dot{v}_2 \end{cases}$$

By the Laplace transformation, the following equations can be derived.

$$\begin{cases} (V_1 - V_2)'' = (1+(n+1)\eta)s(V_1 - V_2) \\ (V_1 + nV_2)'' = s(V_1 + nV_2) \end{cases}$$

The solutions of the above equations are expressed as follows with the introduction of γ_1 and γ_2.

$$\begin{cases} V_1 + nV_2 = A'e^{\gamma_1 x} + B'e^{-\gamma_1 x} \\ V_1 - V_2 = C'e^{\gamma_2 x} + D'e^{-\gamma_2 x} \end{cases}$$

$$\gamma_1 = \sqrt{s},\ \gamma_2 = \sqrt{(1+(n+1)\eta)s} = \sqrt{ps}$$

, where A', B', C' and D' are integration constants.

$$\begin{cases} (n+1)V_1 = A'e^{\gamma_1 x} + B'e^{-\gamma_1 x} + nC'e^{\gamma_2 x} + nD'e^{-\gamma_2 x} \\ (n+1)V_2 = A'e^{\gamma_1 x} + B'e^{-\gamma_1 x} - C'e^{\gamma_2 x} - D'e^{-\gamma_2 x} \end{cases}$$

Then, the following are the general solutions to Eq. (4) in s-domain.

$$\begin{cases} V_1 = Ae^{\gamma_1 x} + Be^{-\gamma_1 x} + nCe^{\gamma_2 x} + nDe^{-\gamma_2 x} \\ V_2 = Ae^{\gamma_1 x} + Be^{-\gamma_1 x} - Ce^{\gamma_2 x} - De^{-\gamma_2 x} \end{cases} \quad (5)$$

A, B, C and D are to be obtained from boundary conditions.

Opposite Direction Drive

In this section, the mode where adjacent lines are driven from the opposite direction is handled. The situation is depicted in Fig. 4. For this mode, analytical expressions turns out to be very complicated if R_t and C_t are not equal to zero. The case where $R_t=C_t=0$, however, gives the worst case scenario in terms of the noise amplitude because the capacitance coupling effect is mitigated if R_t and C_t are finite. Consequently, the $R_t=C_t=0$ case is treated here.

The boundary conditions for this case are as follows (Fig. 4).

$$\begin{cases} V_1'(x=0)=0 \\ V_1(x=l)=0 \\ V_2(x=0)=E/s \\ V_2'(x=l)=0 \end{cases}$$

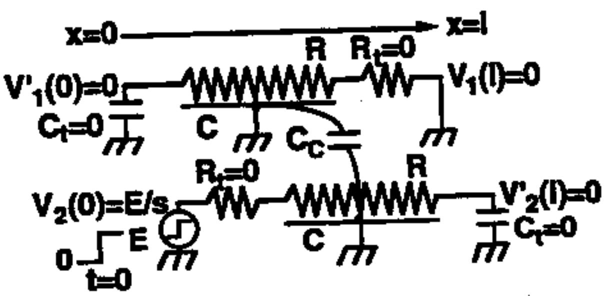

Fig. 4 Adjacent lines are driven from the opposite direction ($R_t=C_t=0$).

Writing these conditions using Eq. (5) yields

$$\begin{cases} A\gamma_1 - B\gamma_1 - nC\gamma_2 + nD\gamma_2 = 0 \\ Ae^{\gamma_1 l} + Be^{-\gamma_1 l} - nCe^{\gamma_2 l} - nDe^{-\gamma_2 l} = 0 \\ A+B+C+D = E/s \\ A\gamma_1 e^{\gamma_1 l} - B\gamma_1 e^{-\gamma_1 l} + C\gamma_2 e^{\gamma_2 l} - D\gamma_2 e^{-\gamma_2 l} = 0 \end{cases}$$

The above linear equations can be solved in terms of A, B, C and D. Then A, B, C and D are substituted back in Eq. (5) and the closed-form expression for $V_1(x,s)$ is obtained. Although the derived expression for $V_1(x,s)$ is very complicated, the peak noise amplitude, v_p, can be calculated using the following initial value theorem of Laplace transform.

$$\frac{v_p}{E} = \frac{v_1(x=0, t=+0)}{E} = \frac{\lim_{s\to\infty} sV_1(x=0,s)}{E}$$

The obtained expression for the peak noise amplitude is simple as follows. The formula is exact. Special case expressions for two-, three- and infinite line systems are also shown.

$$\begin{cases} \dfrac{v_p}{E} = \dfrac{n\sqrt{1+(n+1)\eta} - n}{n\sqrt{1+(n+1)\eta}+1} & \text{(general)} \\ \dfrac{v_p}{E} = \dfrac{\sqrt{1+2C_c/C}-1}{\sqrt{1+2C_c/C}+1} & \text{(two-line)} \\ \dfrac{v_p}{E} = \dfrac{2\sqrt{1+3C_c/C}-2}{2\sqrt{1+3C_c/C}+1} & \text{(three-line)} \\ \dfrac{v_p}{E} = \dfrac{\sqrt{1+4C_c/C}-1}{\sqrt{1+4C_c/C}+1} & \text{(infinite lines)} \end{cases} \qquad (6)$$

If the coupling capacitance, C_C, is equal to the grounding capacitance, C, which can happen in deep submicron designs, v_p/E for two-, three- and infinite line system are 0.27, 0.40 and 0.38 respectively. This means that the noise induced by the coupling would go up to 40% of the signal swing, which may in turn cause malfunction and timing problems. This situation can be verified by the SPICE [6] simulation results as seen in Fig. 5. The SPICE simulation is carried out by using 10 sections of lumped RC blocks.

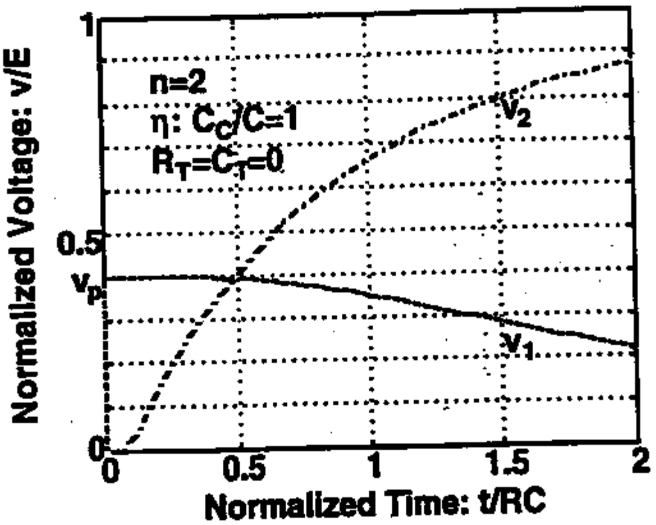

Fig. 5 SPICE simulation results for the case where adjacent lines are driven from the opposite direction (three lines).

Although the expressions for V_1 and V_2 are very complicated, the moments [7-8] of the waveforms can be derived as follows. The Elmore delay of the noise is the coefficient of s in V_1 and the Elmore delay of the perturbed waveform of V_2 is the coefficient of s in V_2.

$$\begin{cases} \dfrac{V_1}{E} = \frac{1}{2} nRC_C s \\ \qquad - \frac{1}{24} nR^2 C_C \{(5n+3)C_C + 8C\} s^2 + O(s^3) \\ \dfrac{V_2}{E} = 1 - \frac{1}{2} R(C_C + C)s \\ \qquad + \frac{1}{24} R^2 \{(3n+5)C_C{}^2 + 10C_C + 5C\} s^2 + O(s^3) \end{cases}$$

For this mode where the adjacent line is driven from the opposite direction, analytical treatment is difficult if the equivalent resistance of driver MOSFET's and the input capacitance of the next gate are to be considered, that is, R_T and C_T are not equal to zero. This case, however, gives the worst case scenario in terms of the noise amplitude because the capacitance coupling effect is mitigated if R_T and C_T are finite.

Same Direction Drive

In this section, the case where adjacent lines are driven from the same direction is treated. The situation is depicted in Fig. 6. For this case, the equivalent resistance of driver MOSFET's and the input capacitance of the next gate can be taken into account in the analysis, that is, R_T and C_T can be finite. The approximation of a driver MOSFET by an equivalent resistor is explained in the Appendix in detail.

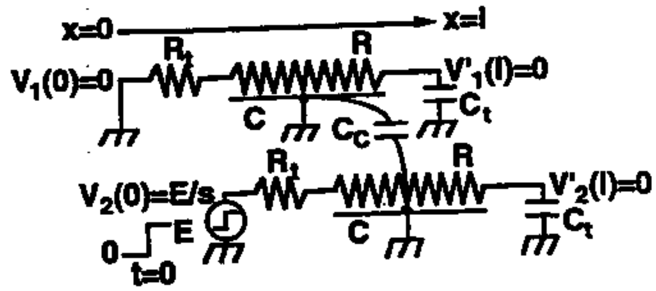

Fig. 6 Adjacent lines are driven from the same direction.

The boundary conditions for this case are written as follow.

$$\begin{cases} -\dfrac{1}{r}\dfrac{\partial v_1}{\partial x}\Big|_{x=0} = \dfrac{-v_1}{R_t}\Big|_{x=0} \\ -\dfrac{1}{r}\dfrac{\partial v_2}{\partial x}\Big|_{x=0} = \dfrac{E-v_2}{R_t}\Big|_{x=0} \\ -\dfrac{1}{r}\dfrac{\partial v_1}{\partial x}\Big|_{x=l} = C_t\dfrac{\partial v_1}{\partial t}\Big|_{x=l} \\ -\dfrac{1}{r}\dfrac{\partial v_2}{\partial x}\Big|_{x=l} = C_t\dfrac{\partial v_2}{\partial t}\Big|_{x=l} \end{cases}$$

If we define $u_1=v_1+nv_2$ and $u_2=v_1-v_2$, u_1 and u_2 give the following equations.

$$\begin{cases} \dfrac{\partial^2 u_1}{\partial x^2} = rc\dfrac{\partial u_1}{\partial t} \\ \dfrac{\partial^2 u_2}{\partial x^2} = prc\dfrac{\partial u_2}{\partial t} = rc\dfrac{\partial u_2}{\partial (t/p)} = rc\dfrac{\partial u_2}{\partial t'} \end{cases} \tag{7}$$

The boundary conditions for u_1 and u_2 are as follows.

$$\begin{cases} -\dfrac{1}{r}\dfrac{\partial u_1}{\partial x}\Big|_{x=0} = \dfrac{nE - u_1}{R_t}\Big|_{x=0} \\ -\dfrac{1}{r}\dfrac{\partial u_2}{\partial x}\Big|_{x=0} = \dfrac{-E - u_2}{R_t}\Big|_{x=0} \\ -\dfrac{1}{r}\dfrac{\partial u_1}{\partial x}\Big|_{x=l} = C_t\dfrac{\partial u_1}{\partial t}\Big|_{x=l} \\ -\dfrac{1}{r}\dfrac{\partial u_2}{\partial x}\Big|_{x=l} = \dfrac{C_t}{p}\dfrac{\partial u_2}{\partial t'}\Big|_{x=l} \end{cases} \tag{8}$$

On the other hand, it is well known that the equation

$$\frac{\partial^2 v}{\partial x^2} = rc\frac{\partial v}{\partial t}$$

with the boundary condition,

$$\begin{cases} -\dfrac{1}{r}\dfrac{\partial v}{\partial x}\Big|_{x=0} = \dfrac{E - v}{R_t}\Big|_{x=0} \\ -\dfrac{1}{r}\dfrac{\partial v}{\partial x}\Big|_{x=l} = C_t\dfrac{\partial v}{\partial t}\Big|_{x=l} \end{cases}$$

has the following solution.

$$\frac{v(l,t)}{E} = 1 + \sum_{k=1}^{\infty} K_k e^{\frac{-\sigma_k \cdot t}{RC}} \approx 1 + K_1 e^{\frac{-\sigma_1 \cdot t}{RC}}$$

This means that $v(l,t)$ can be approximated with single exponential function [2-4]. σ_1 is the pole with minimum absolute value, and K_1 is the corresponding residue and can be approximated as follows.

$$\begin{cases} K_1 \approx -1.01\dfrac{R_T + C_T + 1}{R_T + C_T + \pi/4} \\ \sigma_1 \approx \dfrac{1.04}{R_T C_T + R_T + C_T + (2/\pi)^2} \end{cases} \tag{9}$$

Comparing the above known solution and Eq. (7) and Eq. (8), we can get approximate formulas for u_1 and u_2. v_1 and v_2 are obtained by a linear combination of u_1 and u_2 as follows.

$$\begin{cases} \dfrac{v_1(l,t)}{E} \approx \dfrac{n}{n+1}\left(K_1 e^{-\frac{\sigma_1 t}{RC}} - K_1' e^{-\frac{\sigma_1' t}{pRC}}\right) \\ \dfrac{v_2(l,t)}{E} \approx 1 + \dfrac{1}{n+1}\left(K_1 e^{-\frac{\sigma_1 t}{RC}} + nK_1' e^{-\frac{\sigma_1' t}{pRC}}\right) \end{cases} \tag{10}$$

In the above expression, K_1' and σ_1' are expressed as follows.

$$\begin{cases} K_1' \approx -1.01\dfrac{R_T + C_T/p + 1}{R_T + C_T/p + \pi/4} \\ \sigma_1' \approx \dfrac{1.04}{R_T C_T/p + R_T + C_T/p + (2/\pi)^2} \end{cases}$$

The peak noise amplitude can be derived by searching for the peak value in Eq. (10). This can be achieved by differentiating Eq. (10) and solve $\partial v_1/\partial t = 0$ in terms of t. If we write the solution to this equation as t_p, t_p can be expressed as follows.

$$t_p = \frac{p\ln\left(\frac{K_1\sigma_1}{K_1'\sigma_1'}p\right)}{p\sigma_1 - \sigma_1'} \tag{11}$$

Now, putting t_p back in Eq. (10), the peak noise amplitude is obtained.

$$\frac{v_p}{E} = \frac{n}{n+1}\left[K_1\left(\frac{K_1\sigma_1}{K_1'\sigma_1'}p\right)^{\frac{p\sigma_1}{-p\sigma_1+\sigma_1'}} - K_1'\left(\frac{K_1\sigma_1}{K_1'\sigma_1'}p\right)^{\frac{\sigma_1'}{-p\sigma_1+\sigma_1'}}\right] \tag{12}$$

Several special cases are discussed in the following chapters.

a. Case: R_T, $C_T \gg 1$

In this case, $K_1=K_1'=-1$, $\sigma_1=1/(R_TC_T)$ and $\sigma_1'=p/(R_TC_T)$ hold. Then, from Eq. (12), $v_p \to 0$. This case corresponds to the old situation where interconnection capacitance and resistance are not large compared to MOSFET related resistance and capacitance. For this case, as a matter of course, noise issues can be neglected. The capacitance coupling noise is rather a new headache in VLSI designs.

b. Case: R_T, $C_T \ll 1$

In this case, $K_1=K_1'=-4/\pi$ and $\sigma_1=\sigma_1'=\pi^2/4$ hold.

$$v_p=\frac{4}{\pi}n\left(\frac{1}{1+n_1\eta}\right)^{\frac{1}{n_1\eta}}\frac{\eta}{1+n_1\eta}$$
$$\approx\frac{n\eta}{2+(n+1)\eta}\ \text{(valid when } \eta\le 2)$$

Special case expressions for two-, three- and infinite line systems are shown below.

$$\begin{cases}\dfrac{v_p}{E}=\dfrac{nC_c/C}{2+(n+1)C_c/C} & \text{(general)}\\[2mm] \dfrac{v_p}{E}\approx\dfrac{1}{2}\dfrac{C_c/C}{1+C_c/C} & \text{(two-line)}\\[2mm] \dfrac{v_p}{E}\approx\dfrac{2C_c/C}{1+1.5C_c/C} & \text{(three-line)}\\[2mm] \dfrac{v_p}{E}=\dfrac{C_c/C}{1+2C_c/C} & \text{(infinite lines)}\end{cases}\qquad(13)$$

The approximation is valid when $C_C/C\le 2$. If the coupling capacitance, C_C, is equal to the grounding capacitance, C, which can be happened in deep submicron designs, v_p/E for two-, three- and infinite line system are 0.25, 0.40 and 0.33 respectively. This means that the noise induced by the coupling would go up to 40% of the signal swing, which is the same situation as in the previous chapter. This situation can be verified by the SPICE simulation as seen in Fig. 7.

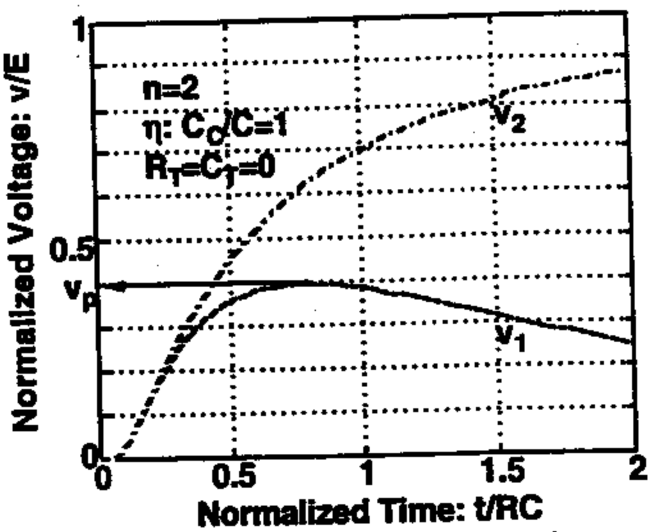

Fig. 7 SPICE simulation results for the case where adjacent lines are driven from the same direction (three lines).

For this case, the time when the noise shows the peak, t_p is approximated as follows.

$$\frac{t_p}{RC}\approx\frac{4}{\pi^2}\frac{p\ln p}{p-1}$$

c. Case: $C_T \ll 1$

This is similar to the previous case and the noise amplitude is approximated as follows.

$$v_p = \frac{R_T+1}{R_T+\frac{\pi}{4}} n \left(\frac{1}{1+n_1\eta}\right)^{\frac{1}{n_1\eta}} \frac{\eta}{1+n_1\eta}$$

$$= \frac{R_T+1}{1.27R_T+1} \frac{n\eta}{2+(n+1)\eta} \text{ (valid when } \eta \le 2)$$

d. Case: $R_T \gg 1$

In this case,

$$v_p = n\left(\frac{1}{1+n_1\eta'}\right)^{\frac{1}{n_1\eta'}} \frac{\eta'}{1+n_1\eta'}$$

$$\approx 0.8 \frac{n\eta'}{2+(n+1)\eta'} \text{ (valid when } \eta' \le 2)$$

, where $\eta' = C_C/(C+Ct)$.

Comparison with Simulation

SPICE simulation is carried out to demonstrate the validity of the noise peak formulas of Eq. (6) and Eq. (12). The simulation results are compared with the calculated results using the analytical formulas in Figs. 8, 9 and 10. As seen from the figures, excellent agreement is observed between the simulated and calculated results.

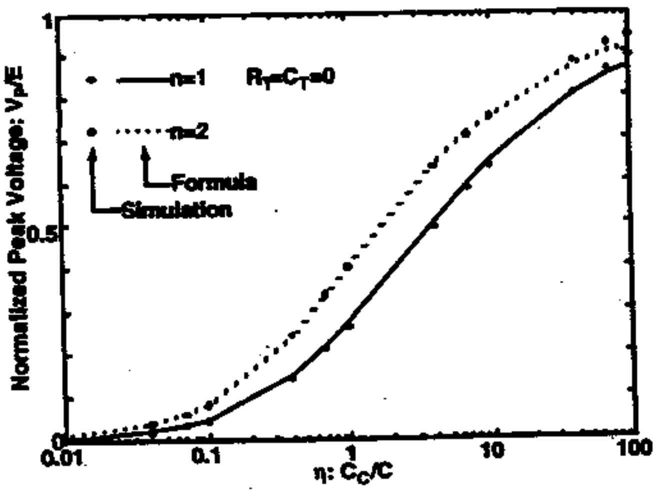

Fig. 8 Simulated and calculated peak noise amplitude using Eq. (6). Adjacent lines are driven from the opposite direction.

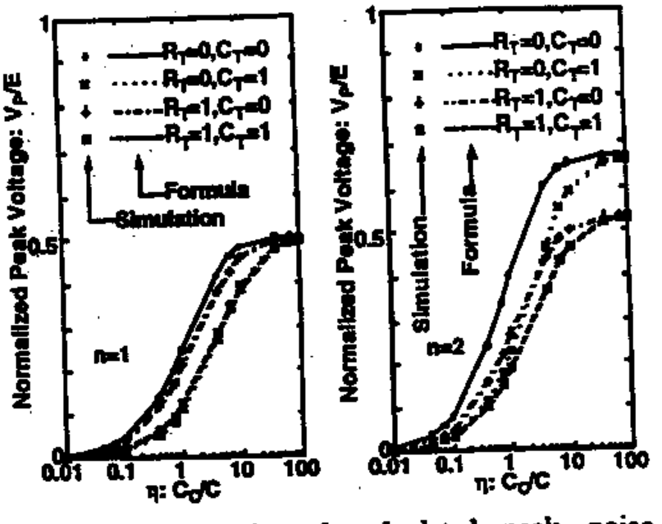

Fig. 9 Simulated and calculated peak noise amplitude using Eq. (12). Adjacent lines are driven from the same direction.

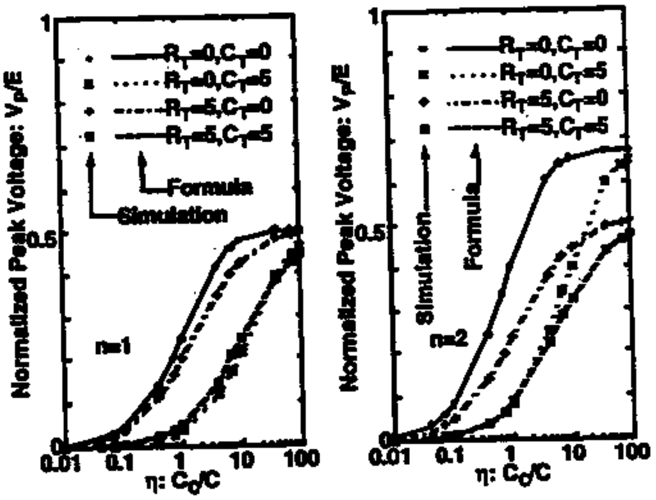

Fig. 10 Simulated and calculated peak noise amplitude using Eq. (12). Adjacent lines are driven from the same direction.

Conclusion

Simple yet useful analytical expressions for peak noise amplitude for capacitively coupled two-, three- and infinite interconnections. The calculated results using the derived formulas of Eq. (6) and Eq. (12) coincide excellently with SPICE simulation results.

In deep submicron VLSI designs where C_C can be comparable to C, the noise induced by the coupling goes up to 40% of the signal swing.

Appendix

Approximation of a driver MOSFET by a resistor

In the text, the driver MOSFET is assumed to be modeled as an equivalent resistor, R_t, and the receiver MOSFET is assumed to be modeled as an equivalent capacitor, C_r. It is well known that the receiver MOSFET can be well represented by a capacitor. The equivalent resistor approximation, however, may need discussion and the topics is touched on in this Appendix.

A typical 0.5μm CMOS case is shown in Fig. A1 where a long RC line is driven by a CMOS inverter. When R_t is greater or equal than R, the behavior of the system is determined by a MOSFET and not by the distributed RC line. The behavior of the RC distributed line is important. Therefore, the following discussion is confine to the case when R is greater than R_t.

Fig. A2(a) shows a driving point voltage, V_d, and a receiving point voltage, V_r. PMOS of the inverter drives the line when the line goes from zero to "High". When R is greater than R_t, V_r follows the slower slope than V_d. Then, PMOS drives the interconnect in the linear region for most of the time as is shown in Fig. A2(b). In this case, R_t can be well approximated as a linear resistor whose resistance is 200Ω.

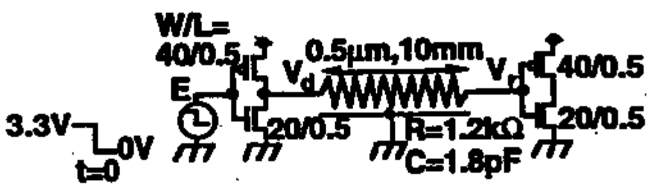

Fig. A1 A long RC line driven by PMOS.

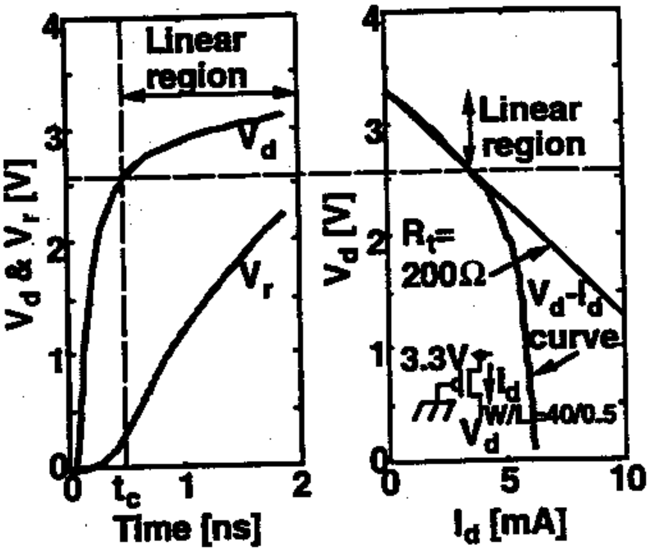

Fig. A2 (a) Voltage waveforms of driving point voltage, V_d, and receiving point voltage, V_r, in case of driven by PMOS. (b) DC characteristics of the driver PMOS.

SPICE simulation is carried out using the resultant equivalent resistor as shown in Fig. A3. Fig. A4 shows waveforms of V_d and V_r with various values of R_t. It is seen that if R_t is chosen as a linear region resistance of PMOS, the equivalent resistor approximation is good.

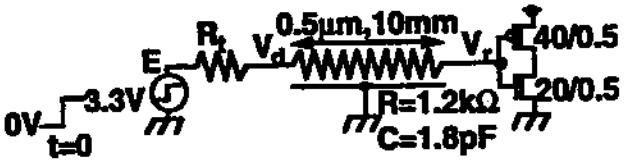

Fig. A3 A long RC line is driven by the equivalent resistance of a driver MOSFET, R_t.

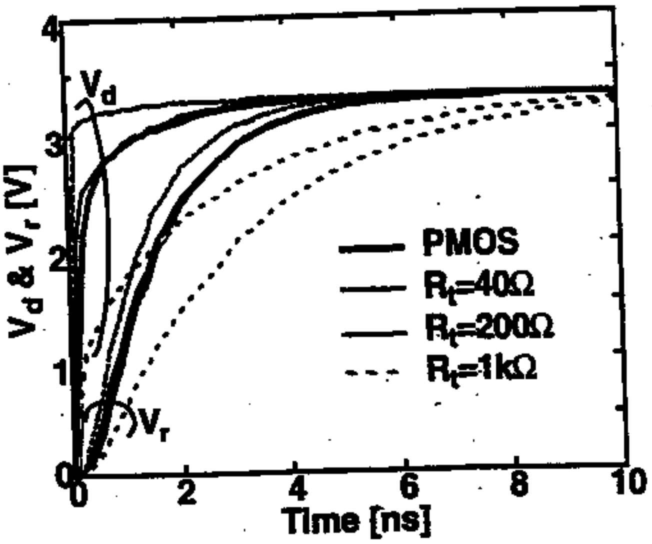

Fig. A4 Waveforms of the driving point voltage, V_d, and receiving point voltage, V_r. The equivalent resistance of driver MOSFET, R_t, is varied as a parameter.

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